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Small-world networks and management science research: a review

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Abstract
This paper reviews the literature on small-world networks in social science and management. This relatively new area of research represents an unusual level of cross-disciplinary research within social science and between social science and the physical sciences. We review the findings of this emerging area with an eye to describing the underlying theory of small worlds, the technical apparatus, promising facts, and unsettled issues for future research.
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Introduction

In the 1920s, Joseph Moreno, a student of Sigmund Freud's was interested in the sources of the 'monsters of the Id.' Breaking from Freud's view that an individual's psycho-emotional problems stemmed from family of origin issues, Moreno believed that they were embedded in contemporary relationships with family, friends, acquaintances, co-workers, and so forth. To measure these connections, he devised a methodology that analyzed a person's network of connections using concepts such as the sociogram, centrality, and isolate. Network analysis later gained popularity in management with the study of small groups, as well as in the landmark, small-world study of large and sparse networks (Milgram, 1967), before it was taken up in the 1960s by organizational sociologists studying career changes (White, 1970a, b), diffusion (Coleman *et al*., 1966), and job search (Granovetter, 1973).

At the time network analysis entered the lexicon of management theory, the dominant theoretical approaches viewed actors as independent units of observation, rather than interdependent and linked parts of a connected whole. The dominant approaches analyzed individual attributes of human capital or organizational and market characteristics rather than a person's social capital or a firm's alliance partnerships (Nohria and Eccles, 1992). While the individual approach remains important, network theory has caught on by showing how issues as wide ranging as creativity, supplier ties, referrals, collaboration, learning, trust, contracts, profits, diffusion, market signaling, entrepreneurship, externalities, price formation, imitation, and production markets can be understood using the principles of network analysis and theory. The evolving field of social network analysis continues to develop with a recent trend exploiting an unprecedented level of cross-talk and collaboration across the physical and social sciences.[1]

One area in which a high level of interdisciplinary research activity has persisted is the topic of small-world networks. Small-world networks correspond to a class of networks in which links among actors are highly clustered, in the sense that on average an actor's connections are also likely to be connected to each other, while the average number of intermediaries needed to connect any two actors across the network, the average path length, remains relatively short. The unique combination of high clustering and short path lengths in the same network along with a growing acknowledgment that small-worlds appear frequently in diverse types of man-made, biological, ecological, and technological systems has suggested that small worlds offer an especially potent organizing mechanism for increasing performance in many different types of systems.

Previous reviews of small-world network have concentrated on the derivations of the methods and techniques used in small-world analysis rather than surveying the empirical findings (Newman, 2000). Other reviews have examined the possibility of using complex networks as a new model for interdisciplinary research on diverse types of interconnected systems (Strogatz, 2001; Amaral and Ottino, 2004), or have explored how the links between small-world networks, scale-free networks, community structure, and network models of dynamic processes such as the spread of disease and social contagion are creating a new science of networks (Watts, 2004).

This review attempts to survey the new literature on small worlds with a focus on social science and management research. We describe the basic methods of small-world analysis, the empirical findings on the relationship between small-world networks and social and economic outcomes, and the unsettled issues for future research. To gain coherence and focus in covering the very large literature in their area, we use empirical studies of real-world networks of interest to social scientists and managerial scholars to bracket our review's coverage. We direct readers interested in the computational details of small-world analysis, or work in non-social science domains such as brain networks or metabolic pathways, and general network analysis to the papers mentioned above and to other reviews (Galaskiewicz, 1996; Borgatti and Foster, 2003; McGrath *et al*., 2003; Sporns *et al*., 2004; Guimera and Amaral, 2005; Cowan *et al*., 2006; Amaral and Uzzi, 2007).

Milgram's small-world study

The idea of a small-world network is typically attributed to the 1967 landmark work of Stanley Milgram. His ideas were based on a series of field experiments that relied on brash creativity to make original discoveries that have had lasting effects on the study of complex networks. Milgram's notion of a small-world network caught the attention of many researchers because it suggested that two characteristics of networks that typically act against each other, clustering and path length, are simultaneously realized in social networks. Milgram's conclusion that a small-world network had a short path length *despite* a high level of clustering – that is, that on average even in a very large small-world network actors are separated by only six degrees of separation or six intermediaries – prompted the speculation that small worlds created unique performance benefits in systems critical for human interaction ranging from creativity to collaboration to communication. This is because the many separate clusters enabled the incubation of a diversity of specialized ideas while short paths allowed ideas or resources to break out of their chambers and mix into new and novel combinations (Uzzi and Spiro, 2005; Fleming and Marx, 2006). Milgram also discovered something much less publicized than the six degrees of separation finding. He found that approximately 60% of the transmissions passed through the same four people! This was a finding worth pondering. It suggested that we are not really all connected to everyone else but rather that there are a few people who are disproportionately well connected and it is through these 'superconnectors' that everyone connects to everyone else. The superconnectors created shortcuts that enabled resources and ideas to hop from cluster to cluster, by passing otherwise long paths from one side of the network to the other. They also made a network potentially fragile to breakup by the removal of just a few superconnectors from the network.[2]

In 2000 the Earth's population surpassed the six billion person mark. In the context of a super-sized world and new communication and organizational devices, Dodds *et al*. (2003) attempted to replicate Milgram's classic small-world study. Did Milgram's insights still hold? If the original Milgram senders and targets had e-mail, could they have made their connections in three or two or one degree of separation? Focusing on e-mail as the medium of transmission, the researchers asked 61,168 participants to deliver messages to 18 targets ranging in location and occupation from a student in Siberia to a Norwegian animal doctor. While just 324 'letters' were successfully delivered, the length of the chains was not dissimilar to Milgram's original findings – most were completed in 5–7 steps. In 1998, mathematicians tested the small-world thesis by estimating how many friends each person on the planet has, and how many friends they have, and so on for the population of the planet (Blakeslee, 1998). They estimated that any two persons chosen at random would indeed on average be separated by about six other persons, jibing again with Milgram's original findings of six degrees of separation.

Small-world theory

Milgram speculated that a small world was a network with a surprisingly few degrees of separation between actors despite the fact that actors tended to have cliquish groups of friends. Watts and Strogatz (1998) showed how Milgram's ideas could be quantified using conventional network measures, and more importantly established that small-world networks constituted a class of networks sharing the joint properties of short path lengths and high clustering.

To quantify a small world, Watts and Strogatz showed that two network measures can be used: average path length (L) and the clustering coefficient (CC). L measures the average number of intermediaries, that is, the degrees of separation, between any two actors in the network along their shortest path of intermediaries. The shorter the average path length, the closer people, resources, or ideas theoretically are to each other in the network. The CC measures how many of an actor's contacts are connected to each other. When many of an actor's contacts are connected to each other, the actor has a highly clustered or cliquish network.

The clustering of a network can be computed as the average of the individual clustering coefficients of each actor, a measure called density (Wasserman and Faust, 1994). Figure 1 shows how a clustering coefficient (CC) is calculated for an actor (dark circle) linked to three other actors who are in turn linked to each other in varying degrees.

Another measure of clustering is called transitivity and it measures the ratio of open to closed triads for the whole network (Holland and Leinhardt, 1971; Feld, 1981; Wasserman and Faust, 1994; Borgatti and Everett, 1999).

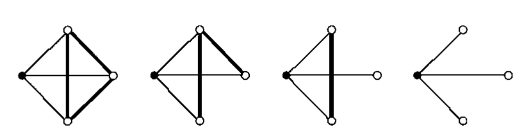

Figure 1 Clustering coefficient.

$$CC = \frac{3 \times \text{number of triangles on the graph}}{\text{number of connected triplets of vertices}}$$

Three configurations yield a triad: A is linked to B who is linked to C; both A and B are linked to C; or both B and C are linked to A. The percentage of closed triads in a network is three times the total number of closed triads (to account for the three possible configurations of triads) divided by the total number of actual triads.

A clustering coefficient varies from 0 to 1. Zero represents no clustering and 1 represents full clustering. A value of 0.65 means that 65% of the triads are closed. The two measures of clustering differ in that the former is unweighted by size and the latter is weighted by size. This means that the former measure shows greater clustering in a network than the latter measure when there are many small ego networks, which by virtue of their size are more likely to have friends of friends connected to each other than are large ego networks.

Once L and CC have been calculated for a network, the question becomes by what standard do we judge a path length to be short and a clustering coefficient to be high? Watts and Strogatz (1998) showed that the relevant comparison was a random graph or network with the same number of actors in it as the observed network but where the links among the actors are made at random. A random network offers a relevant comparison because random networks have relatively short path lengths and low clustering when compared to other classes of networks (Erdös and Rényi, 1960).

In a random network, the likelihood that any pair of actors is linked is given by a constant probability p. For a network with N nodes, this condition implies that the expected degree of an actor (i.e., a person, firm, or other entity) is $k = p(N-1)$. Moreover, if $k < < N$, then the actors connected to actor A will have a probability p of being connected to each other. Thus, node A will have approximately k first-neighbors, k^2 second-neighbors, k^3 third-neighbors, and so forth. If N is much greater than one and if k is at least of order one, then $N = k^L$, where L is the typical minimum path length, that is, the average number of degrees of separation between any two nodes in the network. Solving for L we find that $L = \log N/\mathrm{Log}\, k$, the well-known approximation for the average minimum path length between two nodes in a simple random network. Finally, because $\log N$ increases slowly relative to N, a distinctive property of a random network is a short average path length even for large random networks.

Random graphs also have *low* clustering. In most real-world social networks, many people's friends are also friends of each other. This means that a typical social network has high clustering because the probability of neighboring nodes being connected is relatively higher than the probability of non-neighboring nodes being connected. By contrast, in random graphs neighboring nodes and non-neighboring nodes have the same probability p of being connected. This makes a random graph have low clustering.

Using random networks as their relevant comparison, Watts and Strogatz (1998) showed that a network was a small world if its CC ratio (CC actual $\div$ CC random) was many times greater than 1.0 and its PL ratio (PL actual $\div$ PL random) was approximately 1.0, or if the CC ratio divided by the PL ratio was much greater than 1.0, a measure known as the small world Q (Davis *et al.*, 2003; Uzzi and Spiro, 2005).

Aside from the technical apparatus of using a random graph for comparison, how does one interpret the meaning of a network made up of random ties or a network that has more or less randomness in it? Most persons can think of a few contacts that they have made at random – the person in the adjacent seat on a plane – but this kind of tie making can be rare. Randomness does not necessarily mean that actors make contacts randomly in chance meetings in cafes or the unemployment line but rather that we do not understand, or lack knowledge of, the micro processes that lead to the choice of attachments and that as long as these processes are non-systematic we can treat them as if they were random. Thus, a level of clustering or path length in the network above the level expected at random suggests a persistent rather than a chance structure.

Empirical studies of small worlds

Table 1 presents a summary of small-world research in social science-related studies that have reported the small-world statistics of PL and CC for various networks. The table shows the diversity of studies across three levels of analysis: organizations, persons, and technology. In addition, some studies have reported the change in small-world statistics over a significant period of time revealing how these statistics change with time with the entry and exit of nodes in a population. Other studies collapse time and look at the entire network as one large cross-section. For example, Watt's (1999) analysis of the Hollywood film actors' network looks at connections among actors in the same movie from 1898 to 1997 as one large network. The table shows that most measures of the PL ratio range between 1.0 and 1.5 with a mean of 1.26. By contrast, the CC ratio has a very high variance, ranging from less than one (Baum *et al.*, 2003; Moody, 2004) to 2925.93 for the movie actors network as the networks under study have changed through time and growth.

Drawing together two lines of small-world research on change and robustness, Kogut and Walker (2001) examined how the network of cross-ownership among German firms changed during the 1990s as the German economy internationalized. Despite globalization pressures, which can substantially change ownership cross-holdings among companies, they found that the German network retained its small-world properties of high clustering and short path lengths. Moreover, they demonstrated that a small-world network can preserve its inherent structure despite a substantial number of shocks that rewire ownership links. From 1994 to 1997, they observed that about 101 actual ownership ties were either formed or broken and yet the small world remained, suggesting that small-world networks are robust to even high rates of turnover. Using a simulation that recreated the actual network structure, they randomly rewired two times as many links as had changed in the real-world data. They found that the small-world structure persisted.

This suggests that a massive amount of restructuring is needed to transform a small world into another kind of structure, an important finding for understanding how to

Table 1 Small world studies

Authors	*Network*	*Period*	*N*	*k*	*L Actual*	*L Random*	*CC Actual*	*CC Random*	*Lr*	*CCr*	*Q*
Organizations											
Kogut and Walker (2001)	German firms	1993–1997	291	2.02	5.64	3.01	0.84	0.022	1.87	38.18	20.38
Baum *et al.* (2003)	Canadian I-banks	1952–1957	53	1.36	3.21	4.556	0.023	0.027	0.70	0.85	1.21
		1969–1974	41	2.22	2.82	3.176	0.283	0.054	0.89	5.24	5.90
		1985–1990	142	3.83	2.95	3.144	0.273	0.027	0.94	10.11	10.78
Davis *et al.* (2003)	US Co. interlocks	1982	195	6.8	3.15	2.7	0.24	0.039	1.17	6.15	5.27
		1999	195	7.2	2.98	2.64	0.2	0.039	1.13	5.13	4.54
Verspagen and Duyster (2004)	Strategic alliances*	1980–1996	5504	5.29	4.2	5.25	0.34	0.0008	0.80	425.00	531.25
Schilling and Phelps, (forthcoming)	US alliances in 11 2-digit SIC codes**	1992–2000	171 (157)	3.11 (1.42)	20.39 (18.69)	5.62 (3.01)	0.26 (0.18)	0.04 (0.039)	3.85 (2.84)	10.44 (7.53)	2.71 (2.65)
Persons											
Davis *et al.* (2003)	US Director interlocks	1982	2366	19.1	4.03	2.61	0.91	0.009	1.54	101.11	65.48
		1990	2078	17.4	3.98	2.65	0.89	0.009	1.50	98.89	65.84
		1999	1916	16.3	3.86	2.69	0.88	0.009	1.43	97.78	68.14
Fleming *et al.* (forthcoming)	US patenting inventors***	1986–1990	7069	4.73	2.73	1.14	0.736	0.0452	2.394737	16.28	6.80
Kogut and Walker (2001)	German Co. ownership	1993–1997	429	3.56	6.09	5.16	0.83	0.008	1.18	103.75	87.91
Newman (2004)	Biology co-authorship	1995–1999	1,520,251	18.1	4.6		0.066				
	Physics co-authorship	1995–1999	52,909	9.7	5.9		0.43				
	Mathematics co-authorship	1940–2006	253,339	3.9	7.6		0.15				
Moody, 2004	Sociologists co-authorship	1963–1999	128,151		9.81	7.57	0.194	0.207	1.30	0.94	0.72
		1989–1999	87,731		11.53	8.24	0.266	0.302	1.40	0.88	0.63
Goyal *et al.*	Economists co-authorship	1980–1989	48,608	1.244			0.182				
		1990–1999	81,217	1.672			0.157				
Watts (1999)	Hollywood Film actors	1898–1997	226,000	61	3.65	2.99	0.79	0.00027	1.22	2925.93	2396.85
Smith (2006)	U.S. Rappers		5533		3.9		0.18				
	U.S. Jazz musicians		1275		2.79		0.33				
	Brazilian pop		5834		2.3		0.84				
Technology											
Watts (1999)	Power grids		4941	2.94	18.7	12.4	0.08	0.005	1.51	16.00	10.61
Vazquez *et al.* (2002)	Internet	1997	3112	3.5	3.8		0.18				
		1998	3834	3.6	3.8		0.21				
		1999	5287	3.8	3.7		0.24				

* Chemicals and Electronics Industries, ** average across industries for analysis of separate industries, see Schilling and Phelps, forthcoming.
*** Path length for giant component, **** average for biology, physics, and mathematics.
Empty cells appear when small world statistics were not included in the original article.

measure and gauge the institutionalized power structure behind an industry and economy even if the economy has experienced radical turnover of key players. Similarly, for firms experiencing a high level of turnover either because it is a natural consequence of the business model as in consulting or professional services firms where most recruits are terminated or because of mergers and acquisitions, this result suggests that firms organized as small worlds can experience turnover without disruption to the underlying organization of knowledge transfer and collaboration.

Verspagen and Duysters (2004) examined whether the network of strategic alliances among firms in the chemical and food and electrical industries had small-world properties. Like intercorporate ownership patterns, the network of strategic alliances is another important means by which firms coordinate activity and transfer knowledge. Their data covered 5,504 alliances from 1980 to 1996. Two firms were linked if they had an alliance. Table 1 indicates that this alliance network was a small world with the *PL* ratio close to 1.0 but with a *CC* ratio being much greater than 1.0. While this work did not show a direct connection between the small-world structure and performance, it was important in demonstrating that alliance networks, which are relatively more volatile than ownership ties (Gulati, 2007), have small-world properties and be examined for whether the small-world structure adds values to alliance performance (see Schilling and Phelps, forthcoming for a test).

Looking at a related knowledge transfer and intercorporate coordination problem, Davis *et al*. (2003) examined the stability of the structure of the Fortune 1000 network of corporate directors and the company interlocks of a cohort of 195 F1000 firms from 1982 to 1999. As a source of economic and political coordination (Palmer, 1983; Mintz and Schwartz, 1985; Mizruchi, 1996), Davis *et al*. (2003) were curious about how stable the structure of elites had been relative to the massive restructuring experienced by the US economy. For example, from 1982 to 1999 nearly all of F1000 firms and directors at the network's core had been replaced by a new set. Their results showed that the small-world network of corporate elites remained relatively stable despite the massive turnover of companies and directors. Any two boards remained capable of being linked by no more than just four directors. For example, if Citigroup, a diversified financial wanted to reach the Time Warner board, a media conglomerate, the connector would be Richard D. Parsons. And if Citigroup wanted to connect with Colgate Palmolive, a consumer product company, they could use their link to Time Warner's board to get to Colgate Palmolive through Reuben Mark who links Time Warner and Colgate Palmolive. This work showed that while the individual actors who make up a system can change in terms of capabilities, political interests, technology, or strategy, the underlying organizational structure of a small world continues replicate, suggesting that a small-world network offers a high level of flexibility for organizing a diversity of actors.

In another paper examining the topography and change of a small-world network of economic actors, Baum *et al*. (2003) examined the formation of the short-cut links that connect the clusters of a small world from 1952 to 1990. Looking at the network of Canadian investment banks where a tie exists between two banks if they worked together on a deal, they examined whether the formation of shortcuts was due to '(1) chance partnering of firms in different cliques, (2) intentional partnering by peripheral firms to improve their network positions, or (3) controlled partnering by core firms to maintain their positions.' They found that all three scenarios played a role in explaining the formation of short-cut ties, while chance and strategic partnering played a greater role in their setting. This suggested that the underlying structure of the small world while the product of strategy is also the consequence of chance links that make the complete structure beyond the control of any one firm.

Table 1 also shows that as the Canadian I banking network developed through time, the *PL* ratio remained relatively constant but the *CC* ratio rose by a factor of about 10, suggesting that clustering varies and rises and falls over time even as the path length stays low. To some extent, this makes sense as the *CC* ratio is likely to rise as actors' tenure in the network increases. This is because the likelihood that they will have connections with more actors that are linked to each other increases with time. This study also suggests an important next step in small-world research. Previous research showed that high turnover among German firms as well as US directors, did not substantially change the small-world structure *when* the approximate size of the network remains constant (Kogut and Walker, 2001; Davis *et al*., 2003). Baum *et al*.'s (2003) research suggests that growth combined with turnover has little apparent affect on the *PL* ratio but a significant effect on the *CC* ratio. This suggests that future research on the growth and emergence of real-world small worlds could help reveal how robust these systems are when entry exceed exit or vice versa.

Schilling and Phelps (forthcoming) offered one of the first studies of the relationship between small-world-alliance networks and firm performance in the area of patenting rates. Building on the alliance literature's findings that joint ventures and informal agreements between and among firms can boost a firm's innovation potential (Kogut, 1989; Powell *et al*., 1996; Gulati, 2007; Uzzi, 1997), they reasoned that the more a firm was embedded in an industry-wide alliance network with high clustering and short average path lengths, the more likely it was to gain access to knowledge important for innovation. This is because small-world networks enable reach (links to distant information) while forfeiting little information transmission capacity (efficient information transfer between closely connected firms).

This hypothesis highlighted the difference between small-world theory and the conventional alliance wisdom, which held that 'alliances that create redundant paths within a clique of partners yield transmission capacity but forfeit reach, while alliances that create nonredundant paths to new firms create reach but forfeit bandwidth.' Studying the longitudinal patent performance of 1106 firms in 11 industry-level alliance networks they found results consistent with the small-world hypothesis. The more firms were embedded in networks with small-world properties, the more likely they were to patent at greater rates than firms not in small-world networks. Also, the results were stronger for models employing a two- and three-year lag *vs* a one-year lag, suggesting firms do not quickly realize the innovation benefits of collaboration and that small worlds influence the knowledge base, innovative capacity, and rate of innovation of the firms embedded in them.

Numerous studies have looked at the small-world properties of actor networks. At the boundary between actors that collaborate independently as free-lancers in a market and those that collaborate within the confirms and rules of large firms, Fleming *et al*. (forthcoming) and Fleming and Marx (2006) studied inventors in Silicon Valley and Route 128 in Boston. These inventors collaborated on patented inventions with others within and outside their firms and as freelancers with private patentors or patentors in other firms. Two patentors were linked if they co-authored a patent.

Consistent with predictions, they found that the network of inventors has a small-world structure. Contrary to predictions, they found no relationship between the small-world structure of these networks and patenting rates in these regions. Nevertheless, they did find that short path lengths and the size of the largest component positively correlated with patenting (see also Fleming *et al*. (forthcoming), suggesting that the level of connectivity within the larger subcomponents of a small world does have an effect on performance.

An active area of small-world networks built around freelancers has been done on scientists whose networks of co-authorship can be extensively mapped from historical publication records. Newman (2004) looked at co-authorship networks in biology, physics and mathematics using the medline, Physics E-Prints archive, and mathematical reviews databases respectively. Building on work on the bibliometric properties of published paper archives and databases that have looked at the organizational and institutional properties of collaboration (De Solla Price, 1963), he analyzed these networks using the standard tools of small worlds. He found that all these networks had small-world properties. He found that for the bulk of scientists in these fields gained access to researchers outside their immediate clique of co-authors through a surprisingly small and disproportionately important group of scientists that weave together the invisible college.

Delving deeply into the network structure of a single discipline, Moody (2004) examined the networks of sociologists from 1963 to 1999 using sociological abstracts. Whereas Newman (2004) had looked across fields for similarity, Moody looked within a field's subspecialties for differences. Compared to the physical sciences, co-authorship was rare in sociology. Sixty-seven percent of the papers were sole-authors and 66% of the co-authored papers were written by just two authors (see Wuchty *et al*., 2007 for comparisons of all scientific fields in the physical sciences, social sciences, and arts and humanities from 1955 to 2005). Another surprising finding was that this network, which was made up of many subspecialties such as Marxist theory, economic sociology, criminology, and so forth, was *not* a small world. Presumably, the many criss-crossing subfields of the discipline produced knowledge clusters that were overlapping in content and authorship rather than distinct. 'In each case, the clustering coefficient is a little smaller than random expectations, but distances are significantly greater than expected under random mixing, in direct contraction to the small-world model. These findings suggest that the collaboration network is not composed of a distinct, separate cluster. Instead, permeable theoretical boundaries likely result in a network that folds in on itself, connecting people at greater distances from widely different specialties' (p. 228). Also, in contrast to Newman's (2004) work, Moody found that the high level of overlapping among subfields was not centered on a core of star authors. The network did not fragment until 100% of the scholars with 8–10 links were removed from the network, suggesting that links within and between subdisciplines were not disproportionately dependent on the ties of a few highly published authors with many co-authors.

Goyal *et al*. (2006) also examined the co-authorship network of a single field. They studied co-authorship patterns among economists from 1980 to 1999 and found two notable characteristics of this small world. One, over time the more stable feature of the small world was the increased presence of brokers, 'interlinked stars,' that spanned the network of collaboration (shortening otherwise long path lengths and two) and the average degree of the networks had increased. This suggests, in contrast to Moody's study of sociologists, that co-authorship among economists had become more prevalent with time as the search for knowledge in this small world of many subspecialties has lead authors to work with more collaborators and that it has become more focused on stars that connect the subspecialties.

These findings on the structure of co-authorship networks among scientists show that there is a tendency for a small-world organization, but that it is not universal. Moreover, the similarity of structure that is found between different fields of science may obscure important differences within the internal structure of a field in terms of who are the best connected individuals and the roles they play in weaving together the fields structure. Finally, they suggest that the structure of the network is affected through time by simply the propensity to work in teams on papers. The more a field is oriented towards team *vs* solo production of scientific knowledge, the more a small-world structure appears to arise (see also Guimera *et al*., 2005; Wuchty *et al*., 2007).

Like engineers and research scientists endeavoring to create innovations and new ventures, creative artists must collaborate for access to expertise to make products (e.g., bands are made up of musicians who specialize in the playing of certain instruments) as well as for access to creative material embedded in the conventions of different genres and art forms and personal artist styles (Becker, 1982; Uzzi and Spiro, 2005). Table 1 shows several artists networks. Notably for the large size and diversity of the network, Watts (1999) showed that the network of Hollywood actors (actors were linked if they worked on the same movie) had a small-world structure. Brett Tjaden created the Six Degrees of Kevin Bacon game to examine the network structure of this small world. In this game a player names an actor or actress. If the person acted in a film with Kevin Bacon, then they have a 'Bacon Number' of one. If they acted in a film with someone who has worked with Bacon, they have a Bacon Number of two, and so on. Tjaden showed that the highest Bacon Number is eight, a relatively short maximum path length for a network this large, especially if one considers that Bacon was connected to less than 1% of the actors. The most-connected actor or actress in Hollywood was Rod Steiger. Steiger was highly connected because he worked on film in diverse genres, making him a node that links the diverse clusters of the small world (cited in Uzzi and Spiro, 2005).

Smith (2006) investigated the world of musical artists in Rap (www.allhiphop.com; www.ohhla.com), Jazz, and Brazilian pop music (www.allmusic.com). Table 1 indicates that these networks are diverse in terms of institutional settings, artists backgrounds, and musicians' technical proficiency but all share small-world characteristics of a short path length and high clustering although a definitive conclusion is impossible because the *PL* ratio and *CC* ratio were not reported in his analysis.

Another noteworthy aspect of this study was that it attempted to examine the performance implications of small worlds on the success of the actors in the small world. Measuring the betweenness and degree centrality of rappers he found no evidence in support of an effect of a small-world performance using the number of Gold or Platinum record sales of an artist. Smith (2006) concluded that 'The variables influencing how connected a rapper is can include perceptions of talent, social stature and reputation, and even personal preference. For an example, Dr Dre and Snoop Dogg are both prominent West Coast rappers. Dr Dre only has a node degree of 105 compared to Snoop's 240 despite having a higher record index score [gold and platinum albums] and the same regional roots. [but] …. Dr Dre has gone into more producing but not collaborated as prolifically as Snoop….' Another reason for the zero correlation between small-world characteristics and the success of an artist is that the measure of records sales in terms of Gold and Platinum may be too narrow or selective measures of performance in that few albums reach these marks. Other measures of performance such as number of albums, number of hits, profitability of an album, and so on were not tested due to unavailable data.

Technological networks have been another area of small-world research of interest to management scholarship on technology. Watts (1999) showed that the modern man-made southwestern power grid with about 4900 nodes had a small-world structure. This small-world structure supposedly promoted robustness in the system by buffing functioning parts of the system from parts experiencing failure. This idea of system robustness, a critical aspect of physical networks as well as social networks as noted in the Kogut and Walker (2001) study, is perhaps most noteworthy in the many network studies of the Internet. Vazquez *et al*. (2002) and Barabási *et al*. (2000) have both shown that the high clustering of the routers that make up the Internet along with the few superconnecter routers than tie the Internet together make it highly robust to the random failure of any one node. This line of work has suggested that large-scale technological systems created and maintained to conform to small-world properties can help boost system performance.

Small-world-affiliation networks

In an affiliation or bipartite network, actors work in teams rather than as solo players and each actor on the team is linked to every other actor on the team (Wasserman and

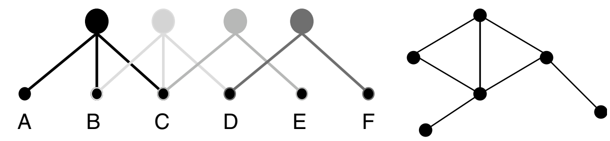

Figure 2 Bipartite-affiliation network and its unipartite projection. *Note*: Top row represents four teams and the bottom row represents the teams' members (e.g., co-authors on a paper or artists that make a show). Teammates are members of a fully linked clique (e.g., A B C, B C D, C E, and D F). Connections form between agents on separate teams when links like (B C) connect the A B C, B C D, and C E teams.

Faust, 1994). Teams are linked if there is (are) a teammate common to both teams. Figure 2 shows an affiliation network's two-tier structure and how it can be projected as an unipartite network.

Affiliation networks deserve special attention for at least three reasons. Affiliation networks are ubiquitous. Many critical types of social networks involve teamwork: actors in a movie (Amaral *et al*., 2000), creative artists who make musicals (Uzzi and Spiro, 2005), organizational project teams, investment bank syndicates, venture capital syndicates (Kogut *et al*., 2007), co-patenting inventors (Gittelman and Kogut, 2003), co-authors on scientific papers (Newman, 2001; Guimera *et al*., 2005), and boards of directors (Robins and Alexander, 2004).

Affiliation networks have been shown to have important effects on performance. They appear to account for major leaps forward in science, art, and philosophical thinking throughout the ages. Going back through all recorded history in Eastern and Western civilizations, Randall Collins' masterpiece, the *Sociology of Philosophies*, examined whether great inventions in art, science, and philosophy occurred by loners or by individuals who were parts of teams, networks, salons, and other community oriented movements. He showed that except for three individuals: Taoist meta physicist Wang Chung, 14th century Zen spiritualist Bassui Tokusho, and 14th century Arabic philosopher Ibn Khaldun, all the other great advances, including Freud, Hegel, de Medici, Smith, Hutchinson, Watson and Crick, and Darwin, came about by individuals who were a part of a network of relationships in which many individuals worked as part of teams.

The statistical properties of affiliation networks differ from the properties of unipartite small-world networks (Borgatti and Everett, 1999; Newman *et al*., 2001). Affiliation small worlds have different levels of clustering and different characteristic path lengths than unipartite graphs. In particular, they have much higher clustering than a unipartite small world because each person's membership on a team means that they are also a member of a fully connected clique (each teammate is linked together, creating a density of 1.0 on each team). Consequently, an affiliation network will appear to have a high level of clustering by virtue of its team topography, not because friends of friends are also friends of each other. Conversely, affiliation networks tend to have shorter path lengths than unipartite networks as the number of overlapping teammates between teams increase.

These differences can be seen in a small-world study by Uzzi and Spiro (2005). They studied the affiliation network of creative artists that made Broadway musicals from the first musical produced in 1877 to the modern day, which included over 400 original shows and more than 2000 creative artists. Each musical was created by a team of artistic specialists (composer, lyricist, choreographer, librettist, director, and producer) who collaborated to produce a musical. Each team formed a fully linked clique. A team linked to the network of past creative artists if at least one artist on the team had worked on a prior team already in the global network.

Figure 3 shows four productions: Pajama Game (1954), West Side Story (1957), Gypsy (1959), and Fiddler on the Roof (1967). Each musical is a made up of a team of five persons. These five musicals can be converted into a unipartite projection in which all the members of the team are connected in a fully connected clique. Across these four musicals, there are several persons that participate in more than one team. In this based-on-real-life example artist **a** is producer Harold Robbins, artist **b** is Stephen Sondheim, the composer in Gypsy and the Lyricist in West Side Story, artist **c** is librettist Arthur Laurents, and artist **d** is director Jerome Robbins. Dynamically assembling these artists through time into one global network, the fully linked cliques link together by artists who worked on more than one team.

The diagram reveals that the *real* level of clustering in an affiliation network is the difference between the *between person clustering* of the actual network and the level of clustering of a random graph of the same size. The reasoning is that all the within team clustering can be explained fully by random rewiring (any rewiring of the links among members of the same team recreates exactly the same pattern of ties within the fully linked clique). Consequently, the real level of clustering in the actual network has to be due to the between clique clustering that cannot be accounted for by random reassignment.

The implications of affiliation network structure on statistical tests of a small world are more clearly seen by looking at how the tests of clustering and path length change. A bipartite small world is still defined as a network that is more highly clustered than a random network and one that has a short path length relative to a random network. However, the randomization process must change to account for the different clustering properties of a bipartite network.

First, the test for comparing the *CC* actual to the *CC* random is biased in an affiliation network because the fully connected teams that make up the network inflate the level of clustering of the actual network relative to the simple random graph comparison used in the unipartite network. Second, the same factors artificially underestimate the actual path length relative to the simple random graph comparison in the unipartite networks. For example, Newman *et al*. (2001) analyzed the small-world statistics of the network of the boards of directors of major US companies (Davis *et al*., 2003) with and without the correction for its bipartite structure. They demonstrated that uncorrected network statistics gave a misleading picture of the true topography of the network. While the uncorrected *CC* ratio showed a very high level of clustering, the network statistics that corrected for the affiliation structure of the network showed that the level of clustering in the network was virtually identical to what would be

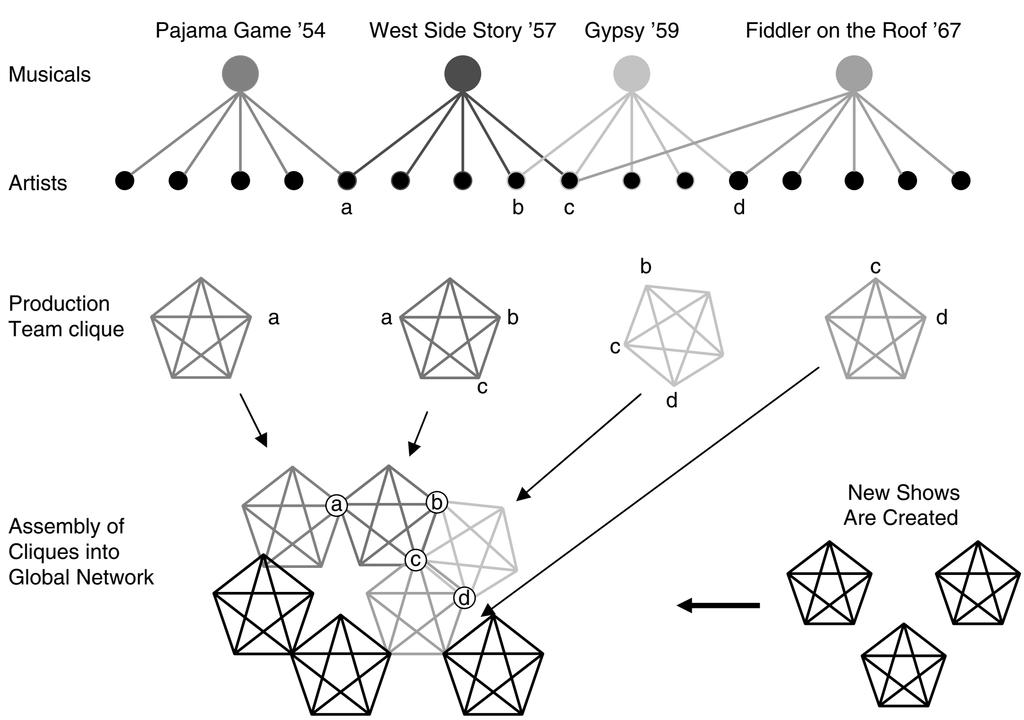

Figure 3 Affiliation networks of broadway creative artists. *Note*: Figure is based on actual data; A = Harold Prince (Producer), B = Steven Sondheim (Composer/ lyricist in Gypsy and Lyricist West Side Story), C = Arthur Laurents (Librettist), and D = Jerome Robbins (Director). As the fully linked cliques are connected to each other through artists who are part of multiple teams, the frequency between clique connections is disproportionately made up of repeated ties and third-party ties. This pattern is illustrated by the high connectivity among the artists who separately worked on West Side Story, Gypsy, and Fiddler and the frequency of the repeated and third ties among B &C and C & D, Sondheim, Laurents and Robbins and Sondheim.

expected in a random bipartite network of the same size. Conyon and Muldoon (2005) found similar results for the boards of director network of UK firms when it was treated as a bipartite rather than a unipartite network. These findings echoed earlier sociological work on boards of directors that showed that the reconstitution of broken board ties between companies could be explained by random reordering (Palmer, 1983; Zajac, 1988; Robins and Alexander, 2004).

Newman *et al.* (2001) obtained an analytical solution for the *random CC* and *random PL* for a bipartite graph. The key adjustment in going from the unipartite to the bipartite network solution is that the random affiliation network has two rather than one distribution of links: the number of individuals per team and the number of teams per individual. Their model is implemented in two steps. First, calculate the tie distributions for each team and each actor in the actual network. Second, for each team and actor in the random graph equivalent, reproduce the degree distribution of the actual network by linking team and teammate nodes randomly. The actual *PL* is calculated by taking the weighted average of the *PL* of each actor in the network. The *PL* for a random bipartite graph is computed by using the same degree distribution as the bipartite random cluster coefficient. In a unipartite random graph, the *PL* is estimated as $\log(n) \div \log(k)$. In the bipartite network, paths are traced from both the perspective of the actor and the team of which the actor is a member.

These differences in methods reflect related differences in the interpretation of the *PL* and *CC* ratio for unipartite and bipartite small worlds. The bipartite *PL ratio* has the same interpretation as in a unipartite network – the greater the *PL ratio*, the greater the mean number of links between actors. The bipartite *CC ratio* has a related but *different* interpretation than the unipartite *CC ratio*. While the *random CC* of an affiliation network lies on a scale of 0–1 and has the same interpretation as the actual *CC*, the *CC* ratio has a different interpretation. It reveals how much between clique clustering there is over that amount expected by simple random assignment. Thus, when the bipartite *CC ratio* is approximately 1.0, there is little between-team clustering. Almost all of the clustering is due to *within-team clustering*. When the *CC ratio* exceeds 1.0, the amount of *between-team clustering* is also increasing – that is, there are more overlapping team memberships that cannot be explained by random linking. Consequently, unlike the *CC* ratio of a unipartite, which should be orders of magnitude greater than the random graph equivalent, the *CC* ratio of an affiliation network can be much smaller and still reveal a significant difference in the level of clustering between the actual and random networks.

As the *CC ratio* rises, the cross-team links increase and are increasingly made up of actors who have previously collaborated (i.e., repeated ties). This occurs because actors that work on multiple project teams are inclined to prefer teammates whom they have worked with in the past or who have worked with others whom they have worked with in the past, a process due to reciprocity and reputation principles (Granovetter, 1985; Uzzi, 1996, 1997).

Table 2 shows the comparison of the bipartite and unipartite small-world statistics for the Broadway Musical creative artists network (Uzzi and Spiro, 2005). Using data

Table 2 Small world affiliation network research

Authors	*Network*	*Period*	*N*	*k*	*L Actual*	*L Random*	*CC Actual*	*CC Random*	*Lr*	*CCr*	*Q*
						Bipartite statistics					
Uzzi and Spiro (2005)	Broadway creative artists	1886	12	2.83	3.71	2.79	0.72	0.87	1.33	0.83	0.62
		1920	597	14.45	3.37	2.88	0.19	0.06	1.17	3.47	2.96
		1945	513	12.62	3.13	2.33	0.29	0.08	1.34	3.73	2.77
		1989	358	7.87	3.60	2.62	0.41	0.18	1.37	2.23	1.62
						Unipartite statistics					
		1886	12	2.83	1.43	2.39	0.72	0.24	0.60	3.05	5.08
		1920	597	14.45	2.89	2.39	0.24	0.02	1.21	9.92	8.20
		1945	513	12.62	3.14	2.46	0.29	0.02	1.27	11.70	9.19
		1989	358	7.87	3.60	2.85	0.41	0.02	1.26	18.50	14.64

Data include the deletion of inactive artists. If an artist was inactive for more than 7 years, the artist and all its ties are removed from the network (See Uzzi and Spiro, 2005 for details).

on Broadway musicals from Ol' Neally, the first show launched in 1877 to the last show produced in 1990, Uzzi and Spiro (2005) reassembled the entire creative artist network. In this network, two creative artists had a link if they worked on the same show together and everyone who worked on the same show were considered teammates. The table indicates the rather large differences that can result from using bipartite *vs* unipartite statistics to measure the same bipartite network. Concentrating on the summary statistic, *Q*, it is apparent from the chart that it is sizably inflated by the unipartite statistics. In 1989, the bipartite *Q* statistic is 1.62 while the unipartite statistic is 14.64, or over nine times as large. This suggests that the unipartite statistics can grossly overstate the degree to which a network is a small world when it is not. Another important difference is the systematically different behavior in the unipartite and bipartite *CC* and *PL* ratios. The unipartite *CC* ratio significantly overstates the amount of clustering in a bipartite network while unipartite *PL* ratio understates the average path length in a bipartite network. Finally, an important difference is the change in the *Q* over time. The unipartite *Q* statistic continues to grow over time while the bipartite *Q* increases and decreases over time. This feature seems to be driven most by the difference in the behavior of the clustering in the network. We see that the random *CC* of the bipartite varies up and down whereas the unipartite *CC* drops and then stays low.

Another empirical finding regarding clustering in bipartite networks showed that it occurs *not* because of persons who are superconnectors, that is, persons who have connections to many others as it does in unipartite small-world networks. Rather, in the bipartite network, team composition plays a dominant role. Robins and Alexander (2004) showed that teams with superconnectors keep the path lengths low and clustering high. They found that the small-world structure of a bipartite network was not due to highly connected individuals working on lots of teams but on teams with many superconnectors. These findings were observed by looking at US and Australia board of director's data – boards that follow different rules and exist in different institutional structures. They looked at interlock data for 229 companies and 489 directors and 198 companies and 225 directors in the US and Australia respectively. Consistent with prior work, they showed that the level of between-team clustering of individual directors was no different than expected by random network of the same size (Newman *et al*., 2001). Directors did not tend to sit on more or fewer multiple boards than expected by chance. However, at the company level of the network, more companies were superconnectors than expected by chance and these companies gained that position by having more than the expected number of directors sitting on their board who sat on more than one board. This suggests that in bipartite networks the small-world properties are not driven by superconnector directors who sit on many boards but rather by boards that attract individuals with multiple board membership. In other words, much of the bipartite structure is dependent on the teams, not individuals, although there is dependence between the two levels in the bipartite structure. This suggests that an affiliation network that cultivates individual star superconnectors without an attempt to accumulate some stars onto similar teams, may enhance an individual directors connectivity at the expense of promoting connectivity across the broader small-world network structure (Robins and Alexander, 2004: 89).

Moving from describing topography and change in a bipartite small-world network to assessing its performance implications, Balconi *et al*. (2004) examined the role of small worlds in the transfer of proprietary knowledge between non-academic and academic patentors from 1978 to 1999 using the EPO-INV database, which is an extract of all Italian patentors' location, professional activities, and social ties. In this network, patentors on the same patent were considered teammates in the affiliation structure. Examining whether the broad small-world hypothesis of information transfer across longs distances holds not only for common knowledge but for the type of proprietary knowledge that flows through patenting networks, they mapped the networks of over 30, 243 Italian patentors to look at the determinants that influence the entry of academic patentors into commercial, non-academic patent networks. Their work provided one of the first large-scale descriptions of patenting networks as well as showed that centrality in the academic patent network was significantly associated with an academic patentor moving into the non-academic network, suggesting that central academics critically affect the flow of proprietary knowledge in this small world.

Uzzi and Spiro (2005) examined the effect of a small world on artists' creativity. Their criteria for creativity were based on two measures of creativity used in show business: (1) profitability and (2) and critics' reviews. The first criterion variable was measured dichotomously (the size of a show's profits is proprietary data) and the second criterion variable was measured as the average of the critics' reviews, which were recorded on a five point Likert scale from pan to rave. As the main independent variables, the authors correlated the *CC* ratio, *PL* ratio, and small world *Q* of this network with the two dependent variables.

They reported several findings about small worlds and performance. First, because the study examined the small-world topography over a long time frame, which included the entry and exit of outstanding talent, as well as many social and economic changes, including two world wars, the Great Depression, the advent of TV and movies, and AIDS, the data could reveal how changes in a small-world-affected performance. They found that a small world changed primarily along two dimensions that paralleled changes in path length and clustering. When there was a low level of *Q*, there were numerous disconnected clusters and the clusters were prone to be linked through one tie. The connecting links also have low cohesion in that they are not disproportionately formed through repeat ties among the actors in the network. At high levels of *Q*, there were few disconnected cliques and many of the cliques were linked by multiple actors who had worked with each other in the past. As the level of *Q* increased, the network became more interconnected and connected by persons who knew each other well because there were more between team links and these links are disproportionately made up of repeat collaborators and collaborators who share third parties in common.

They argued, building on prior research (Granovetter, 1973, Uzzi, 1997; Collins, 1998; Burt, 2004; Fleming and Marx, 2006) that a small world affects the performance of a system as well as the performance of the actors within it in relation to levels of *Q*. If *Q* is low, creative material remains cloistered in the separate teams, an isolating process that is aggravated by the lack of cohesive ties that link separate teams and that promote risk taking on new material among artists. When *Q* increases to a medium level, there is an increase in the level of connections among teams and these connections are increasingly cohesive. Repeated and third-party ties facilitate the cross-team adoption of fresh but unfamiliar creative material. These patterns suggested a linear relationship but one that holds only up to a threshold. After the threshold the effects reverse because too much connectivity and cohesion can undermine the very benefits it creates. If *Q* rises beyond a threshold, the network increases in connectivity and cohesion to a point at which connectivity homogenizes the pool of creative material while cohesive ties promote common information exchanges, limiting the diversity of the pool of creative material and trapping artists in echo chambers of like minded collaborators.

This reasoning suggested a parabolic relationship between a small world and the performance of the actors within it. Consistent with this hypothesis, they found that artists were statistically more likely to produce financially and artistically successful shows at a medium level of *Q* than they were at either high or low levels of *Q*. They also showed that the yearly success of the entire industry varied with the small world *Q*. Seasons with a medium level of *Q*, produced more hit shows and received better artistic notices than did seasons with a high or low level of *Q*. This relationship proved robust for different statistical models, control variables, specifications of a small world (i.e., using *Q* alone or using *CC* ratio and *PL* ratio as separate independent variables).

In a study of political behavior and network structure, Fowler (2005) found a parabolic relationship between voter turnout and the small worldliness of voter's personal networks. He constructed a simulated small-world network using data on 2176 voters surveyed in Huckfeldt and Sprague's 1966 Indianapolis–St. Louis election study. The actual survey showed that voter's personal networks were highly clustered. The probability that a friend is a friend of a friend was 0.61. The probability that two of one's friends talk to each other was 0.47. They did not know the true path length for the political discussion network. They argued that clustering would enhance the influence a committed voter has on the other in their network. They reasoned that as clustering goes up, a committed voter has more paths to influence others in his or her clique. However, they also have fewer connections to the rest of the social network. In their computer simulations as well in regression analyses of actual data they found results consistent with this parabolic hypothesis. Respondents with a mix of in-clique and out-clique friends were 1.5% more likely to vote than people in only in-clique or non-clique networks. Moreover, they found that the parabolic relationship increased as the desire of the committed voter increased. This suggests that participation rates rise with clustering but as clustering reaches high levels in a small world the influence of any single person is more localized.

Guimera *et al*. (2005) constructed the collaboration networks for four academic disciplines – economics, social psychology, astronomy, and ecology, using the co-authorship patterns of the papers written in the top 5–7 journals in each field from approximately 1960 to 1990. In total, the data spanned over 40 years of co-authorship ties among 107, 066 scholars who collaborated on 88,806 papers in the four separate scientific disciplines. They examined the effect of network structure on the scientific impact of the papers written in the separate journal networks of each field in terms of the journal's impact factor, a measure of how intensely the papers in the journal were cited by other papers.

They showed that as the likelihood of teams being made up of incumbents increased, the path lengths of the network decreased relative to a bipartite random graph of the same size. They also showed that as the likelihood of teams being made up of scholars who repeated past relationships increased, the more clustered these networks. This suggests that as the connectivity of the network increases, performance increases. However, as the network becomes highly clustered, performance goes down. If one treats links among incumbents as a measure of connectivity and repeated links as a measure of cohesion, these finding are consistent with Uzzi and Spiro's (2005) findings that low impact factors were associated with co-authors embedded in networks in which connectivity and cohesion were too high or too low whereas high impact factors were associated with co-authors embedded in networks with a medium level of connectivity and cohesion.

Frontiers of small-world analysis

The literature reviewed above leads to several conclusions about small worlds and their implications for social science and management theory. Below are five conclusions that follow from the literature and current trends in research.

Diversity of research

Small-world research in social science and management is surprisingly diverse for a relatively new domain of study. It spans multiple levels of analysis from industries, to firms, to people, and technology and has been conducted by scholars in the physical sciences, social sciences, and arts and humanities to problems of robustness, change, stability, creativity, financial success, political participation, productivity, friendship, and corporate strategy. The new literature also has a focus on the systemic level of analysis, since the study of individual networks can now be connected with the properties of broadly defined network classes which traverse specific contexts and domains. Most prior work on networks was at the egocentric level whereas small-world research is principally on the sociocentric level of analysis of the structure and functioning of the entire network. This shift in emphasis while more a matter of degree than an absolute change in the direction of previous research on networks, does promote the new examination of how systemic level effects create performance outcomes net of individual effects as well as prompts new questions on the relationship between micro behavior and macro structures.

The range of dependent variables appears to be expanding too. While research as focused on the historically significant measures of performance such as creativity, innovation, and M&A activity, new work is likely to continue in this area as well as focus on context specific outcomes. Iravani and Kolfal (2007) for example, used formal models and simulations to show how small-world thinking can be applied to derive the most efficient structure for call center sales force staffing – a special case of the more general operations research issue of efficient queuing. Economists are also becoming more accepting, and even proponents, of networks research in its entirety (Jackson, 2007), an inversion of their initial critical point of view on networks (Arrow, 1998). Because small-world research can influence such a wide range of outcomes variables across many different disciplines, this trend suggests that there is likely to be continued strong growth in small-world research.

Common mechanisms

While research on small worlds is diverse in applications, the literature is driven by a common set of mechanisms (e.g., connectivity/path lengths and cohesion/clustering) that operate for different levels of analysis, contexts, and dependent variables and across diverse methods of research from on-line experiments to field research to simulations to archival research. This characteristic of small-world research is remarkable in that much social science literature has developed different theories and mechanisms for different levels of analysis and different methods of research (Pfeffer, 1982; Hedström and Swedberg, 1998; Davis and Marquis, 2005). For example, transaction cost theory addresses a range of organizational boundary issues but does not attempt to explain organizational or individual creativity, prices, patent rates, scientific impact, Internet robustness, diffusion, learning, job search, or knowledge transfer. In this way, small-world mechanisms stand out as providing an unusually parsimonious set of explanations for many different systems as well as the behavior of the actors embedded within them.

Another aspect of small-world research inviting additional study of mechanisms concerns formation and search. Most research has examined the structure of mature networks where the small-world features of high clustering and short average path lengths can be observed. Comparatively, we know little about how these small worlds arise outside of theoretical models. In other cases, research suggests that new entrants disproportionately link to already highly connected actors but that this process does not hold for incumbents (Barabási, 2002; Powell *et al*., 2005; Kogut *et al*., 2007), suggesting that several formation processes may be at work. Search is another issue of importance. Small-world research presumes that actors can find the short paths. Kleinberg (2000) convincingly showed that small-world networks are only searchable if the actors are structured in a space in which they can conceptualize pathways in terms of geographic space, social space, or some other limiting space. While the participants in the Milgram small-world experiment did make use of these kinds of algorithms (i.e. geographical proximity to the stockbroker, professional proximity to the stockbroker, social proximity to the stockbroker, etc.) it is not yet clear how actors would search a small-world network without these guidelines or how variation in actors' algorithms can be stultified or enhanced by the small-world structure.

Universality

There has been speculation that small worlds appear to be a universal organizing mechanism for social systems (Buchanan, 2002). This review indicates that for the contexts within which small worlds have been analyzed, not *all* social systems are small worlds. For example, Moody (2004) found that the collaboration network of authors who write in sociological journals is not a small world whereas Goyal *et al*. (2006) found that economics is a small world. This suggests that while the range of contexts within which small worlds has been found are remarkable, the initial generalization about their ubiquity may have been overstated, since there can be competing modes of organization. A consequence of this finding may be that the initial research approach of showing that a system is a small world may shift to showing that non-small-world structures can powerfully affect performance. For example, research has shown that scale-free networks and community structures can affect system performance (e.g., Barabási, 2002; Moody and White, 2003; Kogut *et al*., 2007). Currently, however we know little about how global-level network structures compare in their effects on similar systems. Future research is likely to look for essential differences as well as connections between small-world networks, scale-free, community structure and so forth.

Inconsistent results

For studies examining performance, the results have been mixed with small worlds sometimes affecting and sometimes not affecting performance in comparable ways. In other cases, the effects of small worlds in one context do not match the effects of small worlds in another context. For example, Uzzi and Spiro (2005) found a non-linear relationship between small worlds and the financial and artistic performance of creative artists. Fowler (2005) reported a similar association between small worlds and voting participation rates. Consistent with both Uzzi and Spiro (2005) and Fowler (2005), Guimera *et al*. (2005) used different methods to show that too little or too much cohesion or connectivity is associated with lower performance in a small world. By contrast, Schilling and Phelps (forthcoming) found a linear relationship between small worldliness and performance while Fleming *et al*. (forthcoming) and Smith (2006) found indirect or no associations between small-world structures depending on the outcome variable.

Attempting to reconcile these findings is made difficult by the very strength of this new literature: its diverse contexts, long and short time frames of analysis, and differences in performance measurement. For example, Schilling and Phelps (forthcoming) provided several reasons that could account for the fact that their results may have differed from prior work in the literature. They note that while evidence for a parabolic association between small worlds and performance has been found, it has been substantiated only in networks made up of people where high levels of cohesion can demand that performance be traded off for maintaining good personal relations. In their work, they looked at links between corporate actors where issues of interpersonal cohesion may be must less important for collaboration. This suggests that variation in findings across studies could be due to the types of links, levels of analysis or both. Similarly, they looked at their network during a relatively short period of time whereas Uzzi and Spiro (2005), Fowler (2005) and Guimera *et al.* (2005) looked at longer time frames during which levels of clustering path length changed dramatically. This suggests that they may only have observed part of the parabolic relationship, not that it does not exist. In other cases, one can imagine that variation in findings arise because of legal or institutional factors or simply because actor's objectives vary. Smith pointed out that it is not entirely surprising that there was no relationship between a Rapper's position in a small world and the Rapper's album sales because not all Rappers aim for gold records despite being in a position that can enhance their ability to sell more records. Nevertheless, the results suggest that even with the wide variation in contexts and ranges of variable observed, small worlds can affect key performance outcomes beyond what is expected by conventional wisdom. It is likely that future research will aim to reconcile inconsistencies, determine contingent factors, and identify more precise outcome variables.

Dynamics

Although many of the studies of small worlds have used databases that span multiple years of activity or include extremely large numbers of actors, most of the work on small worlds has been cross-sectional. The effects of time and the dynamics of growth and decline are still open questions. Key questions regarding the conditions that give rise to, support, or dismantle a small world, the influence of micro behaviors on macro structures and vice versa, the role of entry or exit, or the interaction between quality of actors and their positions are still largely to be determined. Similarly, there has been little attempt outside the theoretical literature to capture the effect of weighted links, the evolution and change or links, or heterogeneity of links within a small world (Barrat *et al.*, 2004; Sanchez *et al.*, 2007). For example, Watts and Strogatz (1998) showed that a small world can form if just a few long distance links are made at random in an ordered network (each actors has approximately the same number of neighbors). However, while random long distance links help explain how the structure of a small world can arise at a very high level of abstraction, it does not provide a model or explanation of how small worlds evolve socially. Similarly, Guimera *et al.* (2005) look at how the probability of linking to an incumbent *vs* a newcomer or a past collaborator affects the size of the largest group of connected actors in a small-world network (i.e., the giant component). However, a probability of connecting to an incumbent *vs* a newcomer or a repeat relationship is a mechanical analysis. Work still needs to be done on what the probability of linking to an incumbent *vs* a newcomer means in theoretical terms of actual human objectives of quality, trust, status, experience, third parties in common, convenience or other variables known to affect who links with whom (Kogut, 1989; Uzzi, 1997; Kossinets and Watts, 2006; Uzzi and Spiro, 2007; Uzzi *et al.*, 2007). Similarly, they look at the evolution of the giant component but this is just one aspect of a small world's structure and it does not address how the key dimensions of clustering and path length dynamically change. As new ways of conceptualizing and measuring differences in links within networks continues to advance, the dynamic development of small worlds are likely to be influence not just by characteristics of nodes but of links, which could advance our understanding of search, community structure, and performance (Newman, 2001; Barrat *et al.*, 2004). Finally, while it may appear to any individual that following a local strategy is beneficial, such as working with past collaborators, the global effects that result when many actors within the network operate by the same rules may actually hurt the system's the performance and the performance of the actors within it. These links between local and global strategies and their performance implications provide fruitful areas of new research particularly when paired with unique longitudinal data on performance (Kleinberg, 2000).

Conclusion

With the advent of unprecedented amounts of data on the interrelationships among actors in social systems along with new methods for studying large-scale networks, the literature on small worlds has grown rapidly in the social science and management literature. It also represents a unique literature in that it is the product of an exceptional level of interdisciplinary research within the social sciences and between the social sciences and physical sciences. This review has surveyed the empirical literature in this new domain, outlining the unique methods, describing the results, and providing an account of the unsettled issues at the frontier of the field with the aim of promoting future research. Our review shows that the small-world literature is remarkable in the diverse range of outcome variables that have been studied, contexts that have been explored, and levels of analysis that have been treated while at the same time parsimonious in its explanatory variables. Nevertheless, our review finds that small worlds that while small worlds appear to organize many different types of systems in different contexts they are not universal nor is their effect on performance entirely consistent. Small worlds appear to organize the co-authorship networks of diverse disciplines but not all. They also appear to affect performance but not always in the same manner with both complex non-linear as well as linear effects being reported across contexts. Our analysis suggests that these differences may be due in part to the very strength of the literature – its diversity. Because the studies analyzed here span different time frames, the linear effects found in one context may be truncated effects that appear when measuring only sections of a true curvilinear effect that is observable only when longer time frames are analyzed or they may represent true differences that arise due to differences in actors and connections. Our review suggests that these unsettled discrepancies are likely to be resolved through the expansion of research from static to dynamic analyses as well as to analyses that focus not just on the role of the heterogeneity of actors but on the heterogeneity of links between actors. At the same time, we encouraged the continued development of research that is rich in context and highlighting of unique mechanisms that show the relationship between small-world networks and the behavior of the systems as well as the behavior of the actors within it.

Notes

1 At the same time, physical scientists interested in studying economic and social phenomena such as the growth of firms, profitability, and stock market data began to show that there were connections among what had been viewed as dissimilar systems (Watts and Strogatz, 1998; Amaral *et al.*, 1999; Newman and Park, 2003). Before this work, few saw the possible links between metabolic pathways, a biological system; the Internet, a man-made creation; food webs, an ecological system; collaboration and creativity, a psychological and social process; and the spread of infectious diseases, a combination of biological and social processes. All of these complex networks exhibit similar characteristics with respect to certain network properties (Amaral and Ottino, 2004: 147).

2 In a typical experiment, Milgram (1967) chose a target person at random and a set of about 150 or so sender persons at random. In one of his best-known experiments he chose a stockbroker from Cambridge, Massachusetts as his target and 169 senders at random from a small town outside Omaha, Nebraska. These people were diverse – butchers, bakers, homemakers, farmers, clergy, law enforcement officials, sales people, and so on. He sent each of the senders a letter, and in the letter was the name of the single stockbroker. Milgram asked each sender to send the letter back to the stockbroker if he or she happened to know the stockbroker personally. If they did not know the stockbroker, the senders were asked to send the letter to someone they knew personally who could send it directly to the stockbroker or another intermediary and so on until the letter reached the target. Milgram aimed to count the number of intermediaries between two people essentially chosen at random. After he received the letters back from his target person he found that on average it took just six intermediaries to connect two people chosen at random, a number that seemed much lower than most anyone had expected. 'When I asked an intelligent friend of mine how many steps he thought it would take, he estimated that it would require one hundred intermediate persons or more to move from Nebraska to Sharon,' said Milgram. From this remarkable finding, Milgram coined the phrase, 'six degrees of separation,' which has since become part of scientific jargon and popular wisdom.

References

Amaral, Luis A.N. and Julio M. Ottino, 2004, "Complex networks – augmenting the framework for the study of complex systems". *European Physics Journal B*, **38**: 147–162.

Amaral, Luis A.N. and Brian Uzzi, 2007, "Complex systems: A new paradigm for the integrative study of management, physical, and technological systems". *Management Science*, **53**: 1033–1035.

Amaral, Luis A.N., P. Cizeau, P. Gopikrishnan, Y. Liu, M. Meyer, C.K. Peng and H.E. Stanley, 1999, "Econophysics: Can statistical physics contribute to the science of economics?" *Computer Physics Communications*, **121–122**: 145–152.

Amaral, Luis A.N., Antonio Scala, Marc Barthelemy and H. Eugene Stanley, 2000, "Classes of small-world networks". *Proceedings of the National Academy of Sciences*, **97**(21): 11149–11152.

Arrow, Kenneth J., 1998, "What has economics to say about racial discrimination". *Journal of Economic Perspectives*, **12**: 91–100.

Balconi, Margherita, Stefano Breschi and Francesco Lissoni, 2004, "Networks of inventors and the role of academia: An exploration of Italian patent data". *Research Policy*, **33**: 127–145.

Barabási, Albert-László, 2002, *Linked: The new science of networks*. Cambridge, MA: Perseus.

Barabási, Albert-László, Reka Albert and H. Jeong, 2000, "Scale-free characteristics of random networks: The topology of the world-wide web". *Physica A*, **281**: 69–77.

Barrat, Alain, Marc Barthélemy and Alessandro Vespignani, 2004, "Weighted evolving networks: Coupling topology and weight dynamics". *Physical Review Letters*, **92**: 701–704.

Baum, Joel A.C., Andrew V. Shipilov and Tim J. Rowley, 2003, "Where do small worlds come from". *Industrial and Corporate Change*, **12**: 697–725.

Becker, Howard, 1982, *Art worlds*. Berkeley, CA: University of California.

Blakeslee, Sandra, 1998, "Mathematicians show how big world becomes small one". *The New York Times*, Science (6/16/98), p. B13.

Borgatti, Stephen P. and Martin G. Everett, 1999, "Models of core/periphery structures". *Social Networks*, **21**: 375–395.

Borgatti, Stephen P. and Pacey C. Foster, 2003, "The network paradigm in organizational research: A review and typology". *Journal of Management*, **29**: 991–1013.

Buchanan, Mark, 2002, *Nexus: Small worlds and the groundbreaking science of networks*. New York, NY: WW Norton & Company.

Burt, Ronald S., 2004, "Structural holes and good ideas". *American Journal of Sociology*, **110**: 349–399.

Coleman, James S., Elisha E. Katz and Herbert Menzel, 1966, *Medical innovation*. New York: Bobbs-Merrill.

Collins, Randall, 1998, *The sociology of philosophies: A global theory of intellectual change*. Cambridge, MA: Harvard University Press.

Conyon, Martin J. and Mark R. Muldoon, 2005, "The small world network structure of boards of directors", University of Manchester, School of Mathematics, Working Paper MIMS Eprint 2005.15.

Cowan, Robert, Nicolas Jonard and Jean-Benoit Zimmermann, 2006, "Evolving networks of inventors". *Evolutionary Economics*, **16**(1–2): 155–174.

Davis, Gerald F. and Christopher Marquis, 2005, "Prospects for organization theory in the early twenty-first century: Institutional fields and mechanisms". *Organization Science*, **16**: 332–343.

Davis, Gerald F., Mina Yoo and Wayne Baker, 2003, "The small world of the American corporate elite: 1982–2001". *Strategic Organization*, **3**: 301–326.

de Solla Price, Derek, 1963, *Little science, big science*. New York: Columbia University Press.

Dodds, Peter S., Roby Muhamad and Duncan J. Watts, 2003, "An experimental study of search in global social networks". *Science*, **301**: 827–829.

Erdös, Paul and Alfréd Rényi, 1960, "On the evolution of random graphs". *Publication of the Mathematical Institute of the Hungarian Academy of Sciences*, **5**: 17–61.

Feld, Scott, 1981, "The focused organization of social ties". *American Journal of Sociology*, **86**: 1015–10 35.

Fleming, Lee and Matt Marx, 2006, "Managing creativity in small worlds". *California Management Review*, **48**(4): 6–27.

Fleming, Lee, Charles King, III and Adam Juba, forthcoming, "Small worlds and regional innovation". *Organization Science*.

Fowler, James H, 2005, "Turnout in a small world". In A. Zuckerman (ed.) *Social logic of politics*. Philadelphia, PA: Temple University Press.

Galaskiewicz, Joseph, 1996, "The 'new network analysis' and its application to organizational theory and behavior". In D. Iacobucci (ed.) *Networks in marketing*. Thousand Oaks, CA: Sage Publications, pp. 19–31.

Gittelman, Michelle and Bruce Kogut, 2003, "Does good science lead to valuable knowledge? Biotechnology firms and the evolutionary logic of citation patterns". *Management Science*, **49**(4): 366–382.

Goyal, Sanjeev, Marco J. van der Leij and Jose Luis Moraga-Gonzalez, 2006, "Economics: An emerging small world". *Journal of Political Economy*, **114**: 403–412.

Granovetter, Mark, 1973, "The strength of weak ties". *American Journal of Sociology*, **78**: 1360–1380.

Granovetter, Mark, 1985, "Economic action and social structure: The problem of embeddedness". *American Journal of Sociology*, **91**: 481–510.

Guimera, Roger and Luis A.N. Amaral, 2005, "Cartography of complex networks: Modules and universal roles". *Journal of Statistical Mechanics, Theory, and Experiments*, **2**: 02001.

Guimera, Roger, Brian Uzzi, Jarrett Spiro and Luis A. Nunes Amaral, 2005, "Team assembly mechanisms determine collaboration network structure and team performance". *Science*, **308**: 697–702.

Gulati, Ranjay, 2007, *Managing network resources*. London: Oxford.

Hedström, Peter and Richard Swedberg, 1998, *Social mechanisms: An analytical approach*. Cambridge, UK: Cambridge University Press.

Holland, Paul W. and Samuel Leinhardt, 1971, "Transitivity in structural models of small groups". *Comparative Groups Studies*, **2**: 107–124.

Iravani, Seyed M. and Bora Kolfal, 2007, "Call center labor cross-training: It's a small world after all". *Management Science*, **53**: 1102–1112.

Jackson, Matthew O., 2007, "The study of social networks in economics". In J. Podolny and J.E. Rauch (eds.) *The missing links: Formation and decay of economic networks*. New York, NY: Russell Sage.

Kleinberg, Jon, 2000, "Navigation in a small world – It is easier to find short chains between points in some networks than others". *Nature*, **406**: 845.

Kogut, Bruce, 1989, "The stability of joint ventures: Reciprocity and competitive rivalry". *The Journal of Industrial Economics*, **XXXVIII**: 183–198.

Kogut, Bruce and Gordon Walker, 2001, "The small world of german corporate networks in the global economy". *American Sociological Review*, **66**: 317–335.

Kogut, Bruce, Pietro Urso and Gordon Walker, 2007, "The emergent properties of a new financial market: American venture capital syndication from 1960 to 2000". *Management Science*, **53**: 1181–1198.

Kossinets, Gueorgi and Duncan J. Watts, 2006, "Empirical analysis of an evolving social network". *Science*, **311**: 88–90.

McGrath, Cathleen, David Krackhardt and Jim Blythe, 2003, "Visualizing complexity in networks: Seeing both the forest and the trees". *Connections*, **25**(1): 37–47.

Milgram, Stanley, 1967, "The small world". *Psychology Today*, **2**: 60–67.

Mintz, Beth and Michael Schwartz, 1985, *The power of structure of American business*. Chicago: The University of Chicago Press.

Mizruchi, Mark S, 1996, "What do interlocks do? An analysis, critique, and assessment of research on interlocking directorates". *Annual Review of Sociology*, **22**: 271–298.

Moody, James, 2004, "The structure of a social science collaboration network: Disciplinary cohesion from 1963–1999". *American Journal of Sociology*, **69**: 213–238.

Moody, James and Douglas R. White, 2003, "Social cohesion and embeddedness: A hierarchical conception of social groups". *American Sociological Review*, **68**: 103–127.

Newman, Mark E.J., 2000, "Models of the small world". *Journal of Statistical Physics*, **101**: 819–841.

Newman, Mark E.J., 2001, "The structure of scientific collaboration networks". *Proceeding of the National Academy of Sciences*, **98**: 404–409.

Newman, Mark E.J., 2004, "Who is the best connected scientist? A study of scientific coauthorship networks". In E. Ben-Naim, H. Frauenfelder and Z. Toroczkai (eds.) *Complex networks*. Berlin: Springer, pp. 337–370.

Newman, Mark E.J. and Juyong Park, 2003, "Why social networks are different from other types of networks". *Physical Review E*, **68**: 1–9.

Newman, Mark E.J., Steven H. Strogatz and Duncan J. Watts, 2001, "Random graphs with arbitrary degree distributions and their applications". *Physical Review E*, **64**(026118): 1–17.

Nohria, Nitin and Robert G. Eccles, 1992, *Networks and organizations: Structure, form, and action*. Boston, MA: Harvard Business School Press.

Palmer, Donald, 1983, "Broken ties: Interlocking directorates and intercorporate coordination". *Administrative Science Quarterly*, **28**: 40–55.

Pfeffer, Jeffrey, 1982, *Organizations and organization theory*. Boston, MA: Pitman Press.

Powell, Walter W., Kenneth W. Koput and Laurel Smith-Doerr, 1996, "Interorganizational collaboration and the locus of innovation: Networks of learning in biotechnology". *Administrative Science Quarterly*, **41**: 116–145.

Powell, Walter, Douglass White, Jason Owen Smith and Ken Koput, 2005, "Network dynamics and field evolution: The growth of inter-organizational collaborations in the life sciences". *American Journal of Sociology*, **110**: 1132–1205.

Robins, Gary and Malcolm Alexander, 2004, "Small worlds among interlocking directors: Network structure and distance in Bipartite graphs". *Computational & Mathematical Organization Theory*, **10**: 69–94.

Sanchez, Serguei, Felix Reed-Tsochas and Brian Uzzi, 2007, "Degree-degree correlations, assortative mixing, and the robustness of networks. Working paper, Oxford University.

Schilling, Melissa and Corey C. Phelps, forthcoming, "Interfirm collaboration networks: The impact of small world connectivity on firm innovation". *Management Science*.

Smith, Reginald D., 2006, "The network of collaboration among rappers and its community structure". *Journal of Statistical Mechanics*, **2006**: PO2006.

Sporns, Olaf, Dante R. Chialvo, Marcus Kaiser and Claus C. Hilgetag, 2004, "Organization, development and function of complex brain networks". *Trends in Cognitive Science*, **8**(9): 418–425.

Strogatz, Steven, 2001, "Exploring complex networks". *Nature*, **410**: 268–276.

Uzzi, Brian, 1996, "The sources and consequences of embeddedness for the economic performance of organizations". *American Sociological Review*, **61**: 674–698.

Uzzi, Brian, 1997, "Social structure and competition in interfirm networks: The paradox of embeddedness". *Administrative Science Quarterly*, **42**: 35–67.

Uzzi, Brian and Jarrett Spiro, 2005, "Collaboration and creativity: The small world problem". *American Journal of Sociology*, **111**: 447–504.

Uzzi, Brian and Jarrett Spiro, 2007, "What keeps a small world small: Random, natural, and targeted deletion of links. Working paper, Northwestern University.

Uzzi, Brian, Roger Guimera, Jarrett Spiro and Luis Amaral, 2007, "Status and cohesion in the formation of networks: Small worlds, communities, and roles. Working paper, Northwestern University.

Vazquez, Alexei, Romualdo Pastor-Satorras and Alessandro Vespignani, 2002, "Large-scale topology and dynamic properties of the internet". *Physical Review*, **E65**: 066130.

Verspagen, Bart and Geert Duysters, 2004, "The small worlds of strategic technology alliances". *Technovation*, **24**: 563–571.

Wasserman, Stanley and Katherine Faust, 1994, *Social network analysis: Methods and applications*. Cambridge: Cambridge University Press.

Watts, Duncan J., 1999, "Networks, dynamics, and the small-world phenomenon". *American Journal of Sociology*, **105**: 493–527.

Watts, Duncan J., 2004, "The 'new' science of networks". *Annual Review of Sociology*, **30**: 243–270.

Watts, Duncan J. and Steven H. Strogatz, 1998, "Collective dynamics of 'small-world' networks". *Nature*, **393**: 440–442.

White, Harrison C., 1970a, *Chains of opportunity; System models of mobility in organizations*. Cambridge, MA: Harvard University Press.

White, Harrison C., 1970b, "Search parameters for the small world problem". *Social Forces*, **49**: 259–264.

Wuchty, Stefan, Benjamin F. Jones and Brian Uzzi, 2007, "The increasing dominance of teams in the production of knowledge". *Science*, **316**: 1036–1039.

Zajac, Edward, 1988, "Interlocking directorates as an interorganizational strategy". *Academy of Management Journal*, **31**: 428–438.